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A DISTRIBUTED OPTIMAL SOLUTION OF ECONOMIC DISPATCH PROBLEM

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Abstract

Economic dispatch (ED) plays a vital role in power system for energy management. In this article traditional ED of power system is modeled in a distributed way by using consensus theory approach. ED is ensured when each generator in the network has identical Lagrange multiplier value (optimal value). Due to this reason incremental cost is adopted as a consensus term and power mismatch is used as convergence term that will allows the solution to converge towards the optimal solution. The proposed model is fully distributed and needs no central controller for achieving ED. Various scenarios are used to study the designed model. Initially, a small scale three bus and then for final validation IEEE-39 bus systems are used. Furthermore, in IEEE-39 bus system algorithm is tested for faulty generator condition and varying load demand. In all of the test cases the designed algorithm converges in very less iterations. Simulation results justified the effective-eness and convergence efficiency of the deigned system.

Keywords: consensus algorithm; economic dispatch; distributed approach; IEEE-39 bus.

1. INTRODUCTION

The power grid of future will be more sophisticated with will have greater involvement of distributed power generators (DGs'), renewable sources (RGs') and distributed energy storage systems [1]. Economic dispatch (ED) is a very important in a power network as it aims to ensure the best possible solution for managing load and demand balance. There are many solutions already proposed for ED solution e.g. gradient method [2], lambda-iteration technique [3], particle swarm method [4]. All these methods solve the ED efficiently, but have strong requirement of central controller for gathering global information in a centralized power network. This is the reason that traditional centralized power grid these methods are relatively more sensitive to single-point-failure problem. On contrary to this future power grid will be more distributed with large number of distributed-controllers for ensuring the optimal dispatch [5]. Because of this reason a lot of research is going on for finding better solution of ED [6]-[8].

Consensus algorithm is a graph theory based distributed approach that has been widely implemented in communication networks, control theory and multi agent systems [9]. Consensus theory is now also been used to solve the traditional ED in a distributed way [10]-[12]. There are some advantages of this algorithm [13]; network security because nodes/agents (in this paper node/agent refers to a bus) only have to share information with adjacent neighbors, no global information sharing hence no need for central controller, no single-point-failure, easier plug-in and plug-out for agents.

In [14] authors solve ED by gradient method but they have assumed that total supply demand mismatch is known to every agent initially which is not possible without a central controller. In [15] authors used “consensus+innovation” technique for ED but, with two decaying terms α and β to force the solution to converge but no optimal solution is achieved for ED. In [16] and [17] authors also made assumption that supply demand mismatch is initially known to all agents which is again not a fully distributed solution.

From the aforementioned problem we propose a fully distributed solution of ED without any assumption of initial information requirements. The algorithm will be fully distributed and local power mismatch is only shared with the adjacent neighbors in the network. Initially, a three-bus system microgrid is used for the basic testing purpose. Then IEEE-39 bus modified transmission system with undirected communication link is used to achieve the optimal solution for ED, ensure supply-demand balance under variable load condition and handle faulty generator. Moreover, generation limits of each DG are also considered during the modeling. In the Fig.1 basic architecture of microgrid with communication and electric power, the network is shown, generation sources can be any renewable or conventional source.

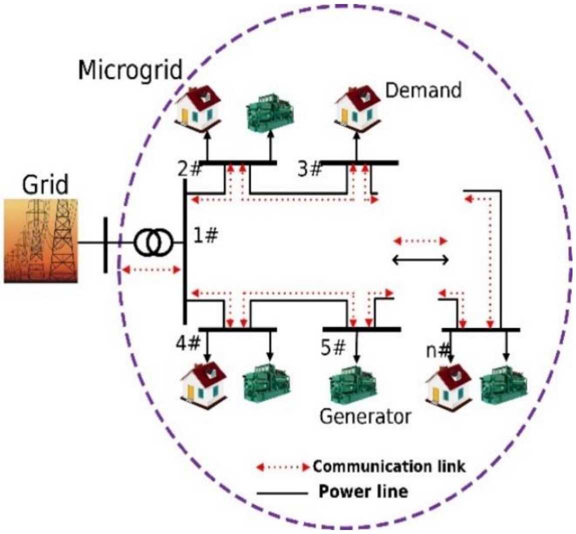

Fig.1. Microgrid Configuration

The remaining paper is organized as follows; section 2 covers basics of graph theory, section 3 discusses ED, section 4 covers modeling of a distributed solution of ED, section 5 is simulation evaluation and section 5 in about conclusions.

2. GRAPH THEORY BASED MODEL

For a better understanding of consensus algorithm first, we will explain graph theory that forms the basis of this algorithm. Let we have a power network with b nodes. The graph $G = (v, \xi)$ represents the inter-connection of nodes, where $v = \{1,2,3 \ldots , b\}$ represents nodes set ato nd $\xi = \{(x, y) \in v\} \subseteq v \times v$ represents edges set (where edge is thethe the link joining two nodes (buses)). Only two adjacent nodes $\{(x, y) \in \xi\}$ can share information with each other. The graph is assumed undirected (as in future grid communication will be two way) also strongly connected. The neighborhood set of node x is denoted by $\Im_x = \{y\,(x, y) \in \xi\}$. The connection of

nodes with each other are denoted by a matrix $\boldsymbol{J} = (\Theta_{xy})_{b\times b}$. The degree of $\Im_x$ for $\{y \in \Im_x\}$ is denoted by $d = \dfrac{1}{\max\limits_{x\in v} |\Im_x| + 1}$ whereas, for $x = y$ is by $d' = \sum\limits_{y\in\Im_x} \dfrac{1}{\max\limits_{x\in v} |\Im_x| + 1}$. Therefore, the entries of matrix $\boldsymbol{J}$ can be defined by Eq. (1).

$$\Theta_{xy} = \begin{cases} d\ , & y \in \Im_x \\ 1 - d', & x = y \\ 0\ , & otherwise \end{cases} \tag{1}$$

The matrix $\boldsymbol{J}$ is row stochastic (sum of each row is 1) and column stochastic (sum of each column is 1).

3. TRADITIONAL SOLUTION OF ED

The idea of ED solution is to achieve optimal dispatch while minimizing fuel cost of generators. The objective function of ED is as:

$$\min \sum_{x=1}^{b} F_x(\rho_x^G) \tag{2}$$

$$\text{s.t.} \qquad \sum_{x=1}^{b} F_x(\rho_x^G) = \sum_{x=1}^{b} F_x(\rho_x^l) = D^l \tag{2a}$$

$$\rho_x^{min} \le \rho_x^G \le \rho_x^{max} \tag{2b}$$

Where, ρ_x^G is power produced by genthe erator, ρ_x^l is local load at node x and D^l total load. $\rho_x^{min}(\rho_x^{max})$ is the minimum (maximum) generation limits of DGs'. Equation (2) is the supply demand equality const-raint. The fuel cost function of a DG is represented as:

$$F_x(\rho_x^G) = \alpha_x(\rho_x^G)^2 + \beta_x\,(\rho_x^G) + \gamma_x \tag{3}$$

where α, β and γ are the cost function coefficients. The incremental cost of x is of the form:

$$\lambda_x = \frac{dF_x(\rho_x^G)}{d\rho_x^G} = 2\alpha_x(\rho_x^G) + \beta_x \tag{4}$$

The optimal ED is achieved when λ of all DGs' are identical to a common value λ'.

$$\lambda' = \frac{D^l + \sum_{x=1}^{b} \frac{\beta_x}{2\alpha_x}}{\sum_{x=1}^{b} \frac{1}{2\alpha_x}} \tag{5}$$

Then by (4) the optimal generation of DG will be:

$$\rho_x^{G-opt} = \frac{\lambda' - \beta_x}{2\alpha_x} \tag{6}$$

This optimal cost is without considering constraint in Eq.(2b). If it is considered then:

$$\begin{cases} \lambda' = 2\alpha_x(\rho_x^G) + \beta_x \,, for \;\; \rho_x^{min} \le \rho_x^G \le \rho_x^{max} \\ \lambda' > 2\alpha_x(\rho_x^G) + \beta_x \,, for \;\; \rho_x^G = \rho_x^{max} \\ \lambda' < 2\alpha_x(\rho_x^G) + \beta_x \,, for \;\; \rho_x^G = \rho_x^{min} \end{cases} \tag{7}$$

If ϕ denotes a subset which contains generation limits of each DG, then we can modify Eq.(7) as:

$$\lambda' = \frac{D^l - \sum_{i\in\phi}^{b} \rho_x^G + \sum_{i\in\phi}^{b} \frac{\beta_x}{2\alpha_x}}{\sum_{i\in\phi}^{b} \frac{1}{2\alpha_x}} \tag{8}$$

In that case the DG's optimal output can be written as:

$$\rho_x^{G-opt} = \begin{cases} \dfrac{\lambda' - \beta_x}{2\alpha_x} & , if \; i \in \phi \\ \rho_x^{min} \; or \; \rho_x^{max} & , if \; i \notin \phi \end{cases} \tag{9}$$

4. DISTRIBUTED SOLUTION OF ED USING CONSENSUS APPROACH

A discrete time consensus theorem from [18] is modified to estimate the value of λ_x as:

$$\lambda_x(k) = \sum_{y\in\Im_x} \Theta_{xy}\lambda_y(k-1) + \eta_x(k-1)\,\Delta\rho_x(k-1) \tag{10}$$

Where, $\lambda_y(k-1)$ ($\Delta\rho_x(k-1)$) is the estimated value of incremental cost (local supply demand mismatch) at k^{th} iteration. And $\eta_x(k-1)$ is control factor and it effects the convergence speed of algorithm. The values of control factor must be very very small ($\eta_x > 0$) to ensure the fast convergence. Whereas, values greater than 1 will result in divergence and hence leading to the non-optimal solution. And combined "$\eta_x(k-1)\,\Delta\rho_x(k-1)$" is feedback-term that allows the solution to converge to optimal value of incremental cost. From estimated $\lambda_x(k)$, estimation can be made for power of each DG $\rho_x^G\,(k)$ by Eq. (4) as:

$$\rho_x^G(k) = \frac{\lambda_x(k) - \beta_x}{2\alpha_x} \tag{11}$$

whereas the estimation of local supply demand mismatch can be defined as:

$$\Delta\rho_x(k) = \sum_{y\in\Im_x} \Theta_{xy}\,\Delta\rho_x(k-1) - \left(\rho_x^G(k) - \rho_x^G(k-1)\right) \tag{12}$$

From Eq.(10)−Eq.(12) defines distributed consensus based ED without constraint Eq.(2b). If it is considered then Eq.(11) will be changed as:

$$\rho_x^G(k) = \begin{cases} \dfrac{\lambda_x(k) - \beta_x}{2\alpha_x} , for \;\; \rho_x^{min} \le \rho_x^G \le \rho_x^{m} \\ \rho_x^{max} \quad , for \;\; \rho_x^G > \rho_x^{max} \\ \rho_x^{min} \quad , for \;\; \rho_x^G < \rho_x^{min} \end{cases} \tag{13}$$

The consensus among network agents will be achieved when following condition is satisfied [18]:

$$\begin{bmatrix} \boldsymbol{\lambda_x}(k) \\ \boldsymbol{\Delta\rho_x}(k) \end{bmatrix} \to \lambda' \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix} as \; k \to \infty$$

That means $\lambda_x(k) \to \lambda'$ and $\Delta\rho_x(k) \to 0$ as $k \to \infty$.

5. SIMULATION RESULTS

In this section designed algorithm is evaluated for different scenarios. First for three DG based microgrid as shown in Fig.2 and then for IEEE-39 bus system. From Eq.(1) matrix $\boldsymbol{J}$ for network shown in Fig.3 can be written as:

$$\boldsymbol{J} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

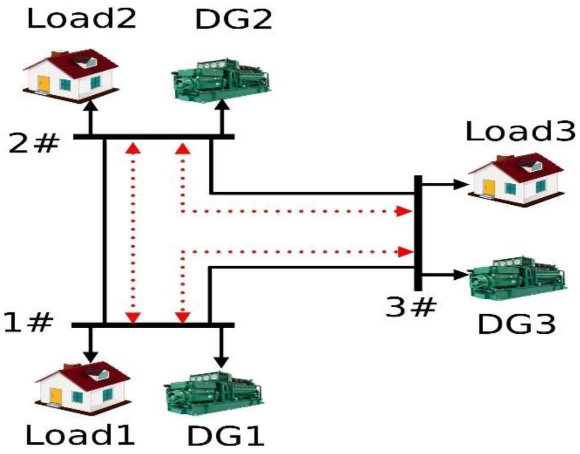

Fig.2 Small scale Microgrid for initial testing

Table 1 shows the cost coefficients and generation limits of 3 DG's and loads in kW.

Table 1. The Parameters of Generators.

DG$_x$	γ_x	β_x	α_x	ρ_x^{min}	ρ_x^{max}	ρ_x^{l}
1	32	1.6	0.087	20	80	30
2	28	2.95	0.056	25	50	40
3	47	3.76	0.065	15	35	60

5.1 Solution of ED for three DG based network

In this section ED is solved for three generators and loads using the algorithm designed in the previous section. The total power demand in the network is 130kW. From Fig.3a it can be seen that the incremental cost converges to the optimal value i.e. 12.09 $/kW in only 11 iterations. Figure.3b shows the values of generated power by each DG; = 60kW, ρ_2^{G-opt}= 40kW and ρ_3^{G-opt}= 30kW respectively. Figure.3c shows the local mismatch of power that finally converges to zero and the total generation matches the demand as shown in Fig.3d.

5.2 Solution of ED for IEEE-39 bus network

In order to test the algorithm for more complex network modified IEEE-39 bus system is used with 10 DG's and 18 loads as shown in Fig.4. Figure.5 shows the undirected communication topology of DG's and loads. The generation and cost parameters of DG's are shown in Table 2. Whereas, total load demand is 2000kW. Matrix $\boldsymbol{J}$ can be derived as done for 3 DGs' based network. Form Fig.6a the algorithm converges to the optimal value of incremental cost by the λ'= 7.90$/kW after 200 iterations. Figure.6b shows the profile of 10 DGs'. Fig.6c and 6d also ensures the power balance in network.

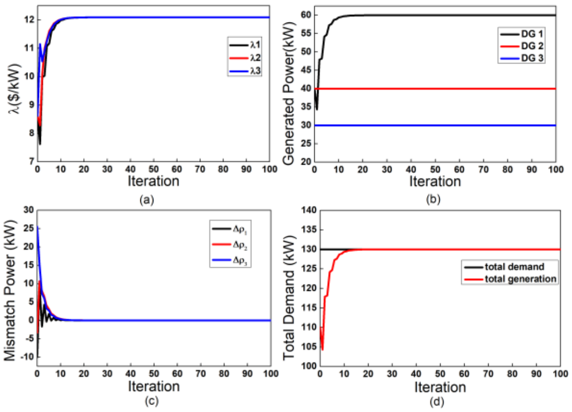

Fig.3 Economic Dispatch for three generators-based system;(a) convergence of incremental cost (b) power generated by each DG (c) local power mismatch (d) generation and demand balance

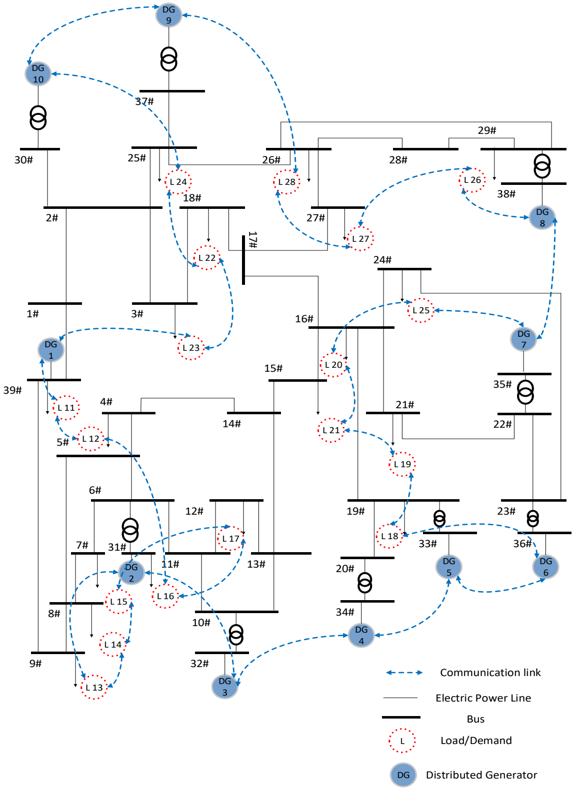

Fig.4 Modified IEEE-39 bus transmission Network with undirected communication links

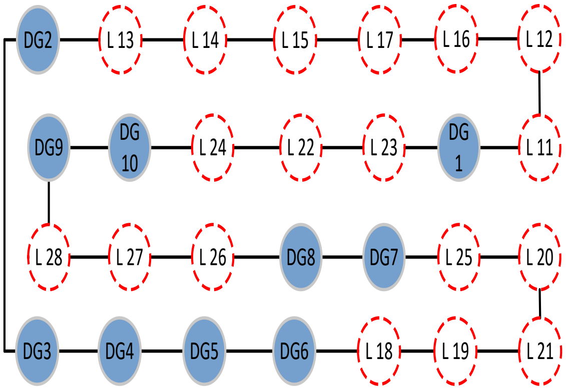

Fig.5 IEEE-39 bus connection topology

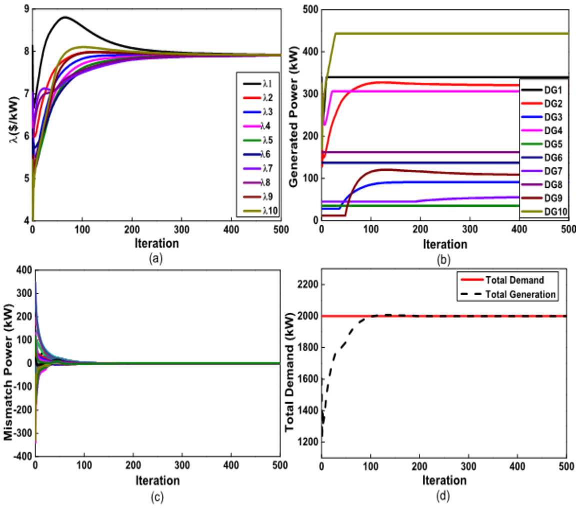

Fig.6 Economic Dispatch for IEEE-39 bus system;(a) convergence of incremental cost (b) power generated by each DG (c) local power mismatch (d) generation and demand balance

5.3 Faulty generator test

In this case, a situation is created for a fault condition at any DG. Suppose, at 500th iteration, DG8 went down (can be seen in Fig.7b) due to some fault/damage. From Fig.7a, it can be seen that the system quickly responds to the situation and remaining DG's rapidly achieve the power balance again and but the incremental cost converges to an increased value i.e. λ' = 8.33 $/kW (cost is increased as remaining DGs' have to generate more power). Figure.7b shows the generation profile of 10 DGs'. Fig.7c and 7d show the power mismatch and supply demand balance. There is slight variation at 500th iteration but sthe ystem quickly retains the stability in few iterations.

Table 2. The Parameters of Generators.

DG$_x$	γ_x	β_x	α_x	ρ_x^{min}	ρ_x^{max}
1	30	5.56	0.0024	60	340
2	25	4.32	0.0056	25	480
3	25	6.60	0.0072	28	290
4	16	3.14	0.0047	40	306
5	6	7.54	0.0091	35	594
6	54	3.28	0.0018	29	137
7	23	7.31	0.0053	45	595
8	15	2.45	0.0063	56	138
9	20	7.63	0.0028	12	165
10	12	4.76	0.0046	30	443

5.4 Time-varying load

In a real-time power system, load demand keeps on varying throughout the day. To consider this situation at 500th iteration load at bus 12 is decreased by 10kW, at bus 21 is increased by 55kW and at bus 27 is increased by 30kW. This results in an increase of total load from 2000kW to 2075kW. But again, demand and generation balance is achieved after a few iterations. From Fig 8a can be seen that λ' is increased to 8.03 from 7.09 $ /kW. Figure.8b shows the generation profile of 10 DGs', can be seen that some DGs have increased their generations as ra esult of ian ncrease in load demand. From Fig.8c and Fig.8d it is evident that mismatch converges to zero and system equalizes to new power demand.

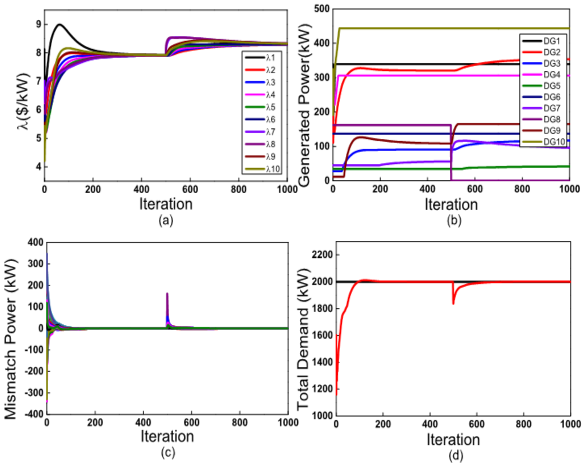

Fig.7 Economic Dispatch with Faulty generator;(a) convergence of incremental cost (b) power generated by each DG (c) local power mismatch (d) generation and demand balance

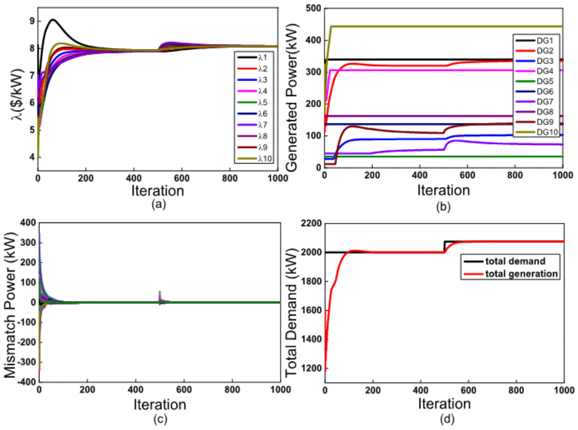

Fig.8 Economic Dispatch with varying load;(a) convergence of incremental cost (b) power generated by each DG (c) local power mismatch (d) generation and demand balance

6. CONCLUSIONS

In this paper, a distributed solution of ED is presented using the consensus theory approach. Initially, a small-scale microgrid with three buses each having a DG and load is used to test the developed model. Further, modified IEEE-39 bus network is used to do final validation by evaluating the performance of algorithm under various test scenarios i.e. considering generation limits of each DG, for fault in any DG, and variable load demand. Under all the test scenarios, the designed algorithm performed efficiently and justified its robustness.

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REFERENCES

[1] U. H. Pourbabak, J. Luo, T. Chen and W. Su," A Novel Consensus-based Distributed Algorithm for Economic Dispatch Based on Local Estimation of Power Mismatch," in IEEE Transactions on Smart Grid, vol. PP, no.99, pp. 1-1.

[2] P. Chen and H. Chang, "Large-scale economic dispatch by genetic algorithm," in IEEE Trans. Power Syst., vol. 10, no. 4, pp. 1919-1926, Nov.1995.

[3] C. E. Lin and G. L. Viviani, \textquotedblleft Hierarchical economic dispatch for piecewise quadratic cost functions\textquotedblright, IEEE Trans. Power App. Syst., vol.6, no. 6, pp.1170-1175, June. 1984.

[4] M. A. Abido, "Optimal design of power system stabilizers using particle swarm optimization," IEEE Trans. Energy Convers., vol. 17, no. 3, pp. 406-413, Sep. 2002.

[5] Y. Xu and Z. Li, "Distributed optimal resource management based on the consensus algorithm in a microgrid," IEEE transactions on industrial electronics, vol. 62, no. 4, pp. 25842592, 2015.

[6] F. Guo, C. Wen, J. Mao and Y. D. Song, " Distributed Economic Dispatch for Smart Grids With Random Wind Power," in IEEE Transactions on Smart Grid, vol. 7, no. 3, pp. 1572-1583, May 2016.

[7] I. Kouveliotis-Lysikatos and N. atziargyriou, " Fully distributed economic dispatch of distributed generators in active distribution networks considering losses," in IET Generation, Transmission \& Distribution, vol. 11, no. 3, pp. 627-636, 2 16 2017.

[8] W. Liu, P. Zhuang, H. Liang, J. Peng, and Z. Huang, " Distributed Economic Dispatch in Microgrids Based on Cooperative Reinforcement Learning, " in IEEE Transactions on Neural Networks and Learning Systems, vol. 29, no. 6, pp. 2192-2203, June 2018.

[9] R. Saber and R. Murray, "Consensus protocols for networks of dynamic agents, "in Proceedings of the 2003 American Control Conference, 2003., vol. 2, pp. 951956, IEEE, 2003.

[10] G. Binetti, A. Davoudi, F. L. Lewis, D. Naso,

and B. Turchiano, \textquotedblleft is tributed consensus-based economic dispatch with transmission losses," IEEE Trans. Power Syst, vol. 29, no. 4, pp. 1711–1720, Jul.2014.

[11] Y. Xu and Z. Li, " Distributed optimal resource management based on the consensus algorithm in a microgrid,"IEEE transactions on industrial electronics, vol. 62, no. 4, pp. 25842592, 2015.

[12] S. Yang, S. Tan, and J. X. Xu, "Consensus-based approach for economic dispatch problem in a smart grid," IEEE Transactions on Power Systems, vol. 28, no. 4, pp. 44164426, 2013

[13] Olfati-Saber, J. A. Fax and R. M. Murray, " Consensus and Cooperation in Networked Multi-Agent Systems," in Proceedings of the IEEE, vol. 95, no. 1, pp. 215-233, Jan. 2007.

[14] W. Zhang, W. Liu, X. Wang, L. Liu, and F. Ferrese, "optimal generation control based on constrained distributed gradient algorithm," IEEE Transactions on Power Systems, vol. 30, no. 1, pp. 3545, 2015.

[15] S. Kar and G. Hug, " Distributed robust economic dispatch in power systems: A consensus + innovations approach, " in Power and Energy Society General Meeting, 2012 IEEE. IEEE, 2012, pp. 18.

[16] S. Yang, S. Tan, and J. X. Xu, " Consensus-based approach for economic dispatch problem in a smart grid," IEEE Transactions on Power Systems, vol. 28, no. 4, pp. 44164426, 2013.

[17] C. Hu, L. Ding, F. Wang, S. Luo, K. Chen and H. Zhang, "Distributed control for economic dispatch problem in a microgrid," 2017 20th International Conference on Electrical Machines and Systems (ICEMS), Sydney, NSW, 2017, pp. 1-6.

[18] M. Mesbahi and M. Egerstedt, Graph-theoretic methods in multiagent networks. Princeton University Press, 2010.